

**ISO TC 20/SC 14**

**ISO/TR 16158**

ISO TC 20/SC 14/WG 3

Secretariat: ANSI

## **Space systems — Avoiding collisions among orbiting objects**

*Élément introductif — Élément central — Élément complémentaire*

### **Warning**

This document is not an ISO International Standard. It is distributed for review and comment. It is subject to change without notice and may not be referred to as an International Standard.

Recipients of this draft are invited to submit, with their comments, notification of any relevant patent rights of which they are aware and to provide supporting documentation.

**Copyright notice**

This ISO document is a working draft or committee draft and is copyright-protected by ISO. While the reproduction of working drafts or committee drafts in any form for use by participants in the ISO standards development process is permitted without prior permission from ISO, neither this document nor any extract from it may be reproduced, stored or transmitted in any form for any other purpose without prior written permission from ISO.

Requests for permission to reproduce this document for the purpose of selling it should be addressed as shown below or to ISO's member body in the country of the requester:

Reproduction for sales purposes may be subject to royalty payments or a licensing agreement.

Violators may be prosecuted.

## TABLE OF CONTENTS

Introduction.....	vii
1 Scope .....	1
2 Symbols and abbreviated terms .....	2
3 Collision avoidance work flow.....	3
3.1 Overview .....	3
4 Perceiving close approaches.....	4
4.1 Orbit data.....	4
4.2 Initial filtering.....	4
4.3 Eliminating infeasible conjunctions.....	4
5 Determining potential collisions for warning and further action (close approach screening).....	5
5.1 Symmetric keepout.....	5
5.2 Bounding volume keepout .....	6
5.3 Probability techniques.....	6
5.4 Maximum probability.....	7
5.5 Bounding volume based on probability .....	8
5.6 Comparison of techniques .....	9
6 Probability of survival .....	9
6.1 Trending.....	10
6.2 Cumulative probability.....	10
6.3 Bayesian assessment .....	11
7 Additional information for judging courses of action.....	12
7.1 Maneuver capability .....	12
7.2 Spacecraft characteristics.....	12
7.3 Quality of underlying orbit data.....	12
8 Consequence assessment .....	12
8.1 Guidance for population risk.....	12
8.2 Traffic impacts .....	13
9 Requirements for warning and information for avoidance.....	13
9.1 Orbit data.....	13
9.2 Minimum data required for warning of and avoiding collisions.....	13
9.3 Optional elements of information .....	14
10 Conjunction and collision assessment work flow and operational concept .....	14

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT) see the following URL: [Foreword - Supplementary information](#)

The committee responsible for this document is ISO/TC 20, *Aircraft and space vehicles*, Subcommittee SC 14, *Space systems and operations*.

## Introduction

This Technical Report describes the work flow for perceiving and avoiding collisions among orbiting objects, data requirements for these tasks, techniques that can be used to estimate the probability of collision and guidance for executing avoidance manoeuvres.

The process begins with the best possible trajectory data, provided by satellite operators or sensor systems developed for this purpose. The orbits of satellites must be compared with each other to discern physically feasible approaches that could result in collisions. The trajectories so revealed must then be examined more closely to estimate the probability of collision. Should a collision be likely within the criteria established by each satellite operator, the spectrum of feasible manoeuvres must be examined.

There are several different approaches to conjunction assessment. All have merits and deficiencies. Most focus on how closely satellites approach each other. This is often very uncertain since satellite orbits generally change more rapidly under the influence of non-conservative forces than observations of satellites in orbit can be acquired and employed to improve orbit estimates. Spacecraft operators require the fullness of orbit data in order to judge the credibility and quality of conjunction perception. This information includes the moment of time of the last elaboration of orbit (the epoch) and the standard time scale employed, state vector value or elements of orbit at this moment of time, the coordinate system description that presents the orbital data, the forces model description that was used for orbital plotting, and information about the estimation errors of the orbital parameters. Essential elements of information for this purpose are specified in ISO 26900.

There are also diverse approaches to estimating the probability that a close approach might really result in a collision. This is a statistical process very similar to weather forecasting. Meteorologists no longer make definitive predictions. They provide the probability of precipitation, not whether it will rain. All conjunction assessment approaches are in some way founded in probabilities. Probability of collision is also a highly desirable element of data. It must be accompanied by metadata that allows operators to interpret the information within their own operational procedures.

How near satellites might be to each other and the probability they might collide if they were that close are only two discriminants of potentially catastrophic events. Since the objective is that the satellite survives despite many potential close approaches, cumulative probability of survival is also important information. Responding precipitously to the close approach nearest at hand might only delay the demise of the satellite or even contribute to a subsequent more serious event. The evolution of orbits toward close approaches and the cumulative probability that a satellite might survive for a period of time are also important.

Finally, the state of each of the conjunction partners, their ability to maneuver or otherwise avoid contact, and the outcomes of past events that are similar guide courses of action.

# Space systems — Avoiding collisions among orbiting objects

## 1 Scope

This Technical Report is a guide for establishing essential collaborative enterprises to sustain the space environment and employ it effectively. This requires diligent collaboration among all who operate satellites.

This Technical Report describes some widely used techniques for perceiving close approaches, estimating collision probability, estimating the cumulative probability of survival, and manoeuvring to avoid collisions.

NOTE Satellite operators accept that all conjunction and collision assessment techniques are statistical. All suffer false positives and/or missed detections. The degree of uncertainty in the estimated outcomes is not uniform across all satellite orbits or all assessment intervals. No comparison within a feasible number of test cases can reveal the set of techniques that is uniformly most appropriate for all.

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO/TR 11233, *Space Systems — Orbit determination and estimation — Process for describing techniques*

ISO 26900, *Space data and information transfer systems — Orbit data messages*

ANSI/AIAA S-131-2010, *Best Practices in Astronautics: Propagation*

AIAA G-043-1992, *Guide to Developing Operational Concepts*

## 3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

### 3.1

#### **conjunction**

apparent meeting or passing of two or more objects in space

### 3.2

#### **collision**

act of colliding; an instance of one object striking another

### 3.3

#### **covariance**

measure of how much variables change together

Note 1 to entry: For multiple dependent variables, a square, symmetric, positive definite matrix of dimensionality  $N \times N$ , where  $N$  is the number of variables.

### 3.4

#### **encounter plane**

plane normal to the relative velocity at the time of closest approach

### 3.5

#### **false alarm**

statistical Type I error, when a statistical test fails to reject a false null hypothesis

### 3.6

#### **ICD**

#### **Interface Control Document**

formal means of describing the inputs and outputs of a system, the interfaces among systems, or the protocols among physical or electronic elements of an entity

### 3.7

#### **operational concept**

roles, relationships, and information flows among tasks and stakeholders and the manner in which systems and processes will be used

## **4 Collision avoidance workflow**

The avoidance process begins with orbit data, the content of which is specified in ISO 26900. The data can be provided by collaborating satellite operators and from observers who are capable of viewing satellites. The nature of each object should also be known if possible. This information includes size, mass, geometry, and the operational state (for example, whether active or inactive). Finally, collision probability should be estimated based on the inevitable imprecision associated with orbit determination and other hypotheses and measurements. Figure 1 depicts this top-level work flow.

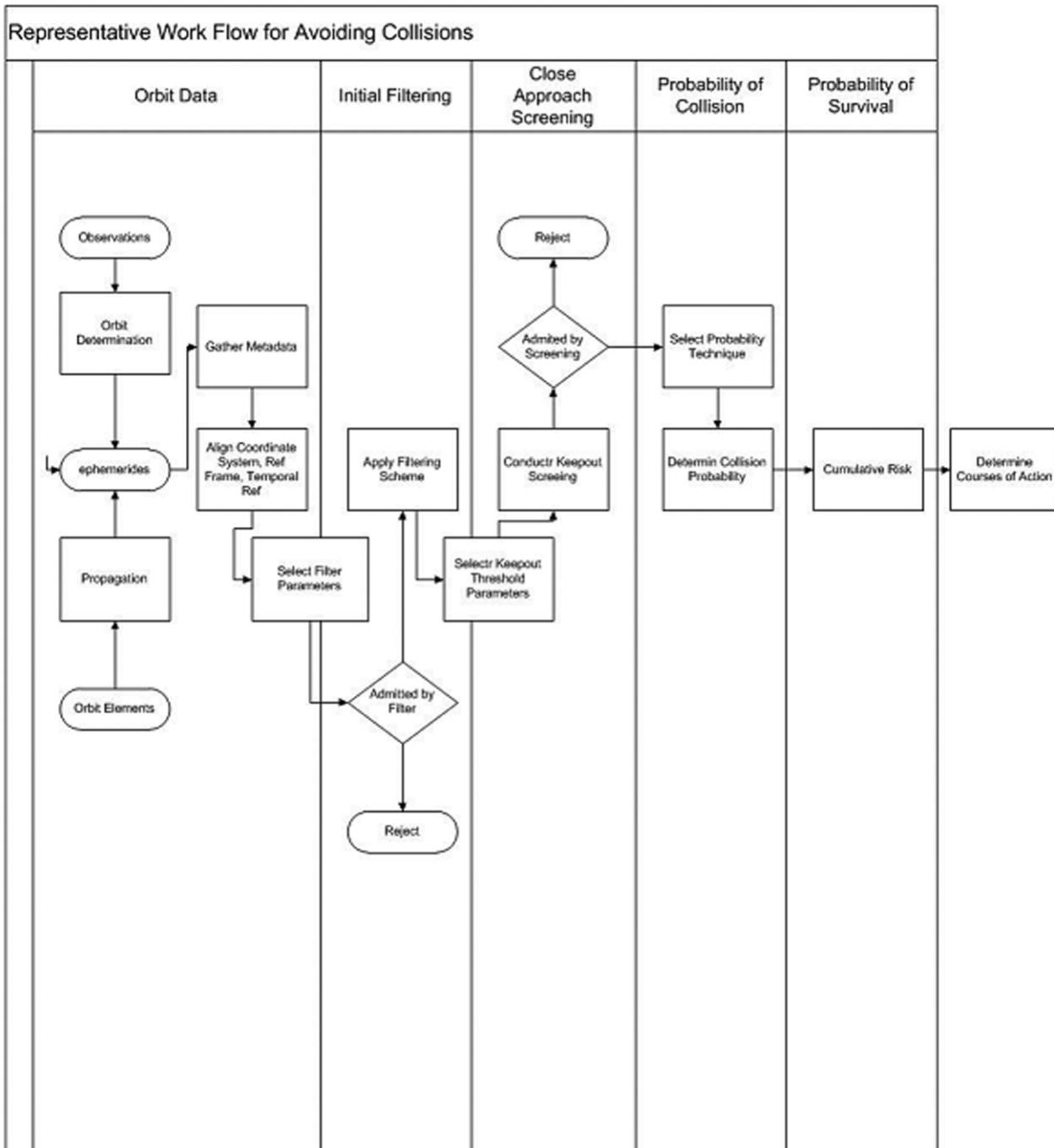


Figure 1 — Top-level collision avoidance work flow

## 5 Perceiving close approaches

### 5.1 Orbit data

#### 5.1.1 Inputs

Inputs to conjunction assessment are principally data that specify the trajectories of the objects of interest. These are one of three types of information: orbit elements, ephemerides, or observations of satellites. Orbit elements in this context include parameters that describe the evolution of the trajectory and which can be used to estimate the trajectory in the future. They are derived from past observations

of satellites. Ephemerides are time-ordered sets of position and velocity within which one interpolates to estimate the position and velocity at intermediate times. Ephemerides should span the future time interval of interest, the equations of motion having been propagated by the provider. Observations are measurements of satellite position and velocity from one or more well-characterized and registered instruments. The recipient must use those observations to estimate the evolution of the trajectory either through direct numerical integration of governing equations or by developing orbit elements for subsequent propagation. ISO 11233 describes the manner in which a provider's orbit determination scheme should be codified. There are normative formats for orbit elements and ephemerides (see ISO 26900). There are no normative formats for transmitting observations.

It is extremely important to realize that trajectory estimates are derived from measurements that cannot be precise such as aspheres. This is why they are called "estimates." The input information must include characterized uncertainties. Uncertainty in any of the independent variables or parameters introduces imprecision in all of the dependent variables that describe the evolution. The appropriate expression of uncertainty is, therefore, a square matrix whose dimension is the number of elements of the state, called a state vector. If uncertainties are not provided or are wrong, one cannot determine properly the probability that two objects might collide.

### **5.1.2 Propagating all orbits over the interval of interest**

All orbits being under consideration shall be forecast in the model in which they were created. Since orbit determination and propagation are uncertain, the propagation scheme must be well suited for this interval. ANSI/AIAA S-131-2010 is a normative reference for orbit propagation. Osculating orbit estimates grow imprecise over time intervals long compared to the time span of underlying observations. This imprecision is sufficient to make collision probabilities misleading. Therefore, conjunction assessment in low Earth orbit is unreliable at the present state of the art for periods longer than approximately one week beyond the latest orbit determination, depending on the orbit of interest. Some particularly stable orbits might be estimated reliably for longer periods. Probability of collision can be estimated over long periods using consistent statistical descriptions of satellite orbits and the evolution of the debris environment. These techniques estimate whether a conjunction will occur or not but cannot expose which specific objects might be involved.

## **5.2 Initial filtering**

### **5.2.1 All against all**

The most complete process would examine each object in orbit against all others over the designated time span. Most techniques eliminate A-B duplication, defined as screening B against A in addition to A against B. Therefore, the number of screenings necessary is not the factorial of the number of satellites.

It is impossible to know how many objects orbit the Earth. Many escape perception. The best a satellite operator can do is to consider those that have been detected. One cannot screen against unknown objects that one estimates might be present.

## **5.3 Eliminating infeasible conjunctions**

Much of the population in orbit physically could not encounter many other satellites during the period of interest. For example, even if uncontrolled, geostationary satellites 180 degrees apart in longitude are not threats to each other.

### **5.3.1 Sieve**

Sieve techniques employ straightforward geometric and kinematic processes to narrow the spectrum of feasible conjunctions based on the minimum separation between orbits. They are based variously on orbit geometry, numerical relative distance functions, and actual orbit propagation. The concept is to

examine proximity of one satellite to another sequentially in parameter space beginning with the parameter that most effectively discriminates separation distance. To account for approximations in orbit analysis, a distance buffer (pad) may be added to the filter screening distance threshold. For example, if in-track separation is likely to be the best indicator of separation, satellites that are far apart in-track need not be screened further cross-track. They differ in computational efficiency and the degree to which close approaches are all perceived. There is no normative approach since different techniques are satisfactory for different satellites and operator judgements.

### **5.3.2 Toroidal elimination**

Toroidal elimination eliminates objects by determining which mean orbits might touch a toroidal volume defined by the orbit of the satellite of interest and a keepout volume cross-sectional area.

### **5.3.3 Apogee-Perigee filters**

This approach eliminates satellites whose apogees are lower than the perigee of the satellite of interest and perigees are sufficiently greater than the apogee of the satellite of interest. The criterion for sufficiency is based either on operator experience or risk tolerance. Risk can be quantified with techniques of signal detection and receiver operating characteristics discussed subsequently. Volumetric screening is of the same nature, eliminating satellites whose orbits are outside the volume of space described by the orbit of the satellite of interest.

### **5.3.4 Statistical errors**

Since each of these techniques relies on trajectory information that is imprecise, these filters will suffer Type I, failure to identify real threats, and Type II errors (including satellites that are not threats). Filter parameter selection should be based on the user's tolerance for both kinds of errors. Every filtering scheme will include events that should be discarded and discard events that should be included.

## **6 Determining potential collisions for warning and further action (close approach screening)**

Initial filtering provides little information for mitigating collisions. The next task is judging whether the actual states of the involved satellites are sufficiently threatening. The first step is determining whether satellites come extremely close to each other. This is the judgement of each satellite operator. It may be based on satellite sizes, the consequences of a collision, the confidence one has in orbit estimates and propagation, and other subjective factors. As with initial filtering, even this more refined level of discrimination will miss some threats. The possibility of false alarms and missed detections increases the farther in the future one extrapolates.

### **6.1 Symmetric keepout**

The most straightforward keepout volume is symmetric. These are easiest to implement but might encompass considerably more than the vulnerable geometry of the satellite. These can be spheres, cubes, or any other three-dimensional volumes of operator-judged size. The satellite of interest may be enveloped symmetrically and osculating orbits of other satellites tested for penetrating the volume. Alternatively, the bounding volumes of both satellites may be screened for intersection. This is generally the most conservative approach, identifying as potential collisions requiring action many events that are extremely improbable.

### **6.2 Bounding volume keepout**

This approach envelops the satellite of interest in a volume that is not symmetric. The volume could be ellipsoidal, a rectangular parallelepiped, or a shape composed of surfaces nearly conformal with the

satellite. The geometry of the bounding volume could be based on operator experience. For example, one might use fairly consistent orbit uncertainties along track, radial from Earth Center, and normal to the plane defined by both of these directions. The volume could also be determined from more exhaustive probabilistic calculations that are too resource intensive to use frequently.

### 6.3 Probability techniques

By definition, the probability that if two objects are separated by a given distance they might actually collide is the volume of the intersection of the objects' position probability densities. It is a function of time.

All satellite orbits are imprecise. Approximations to physical processes (process noise) and imprecise observations of satellite states of motion (measurement noise) lead to imprecise estimates of the future states of satellites. The imprecision is represented by variances and covariances of the dependent parameters among each other. These form a covariance matrix. It represents generally mean squared deviations of estimated (expected) values of each dependent variable from those inferred from measurements. A covariance matrix is by definition symmetric and positive-definite if all of the variables are independent.

When the duration of a conjunction is very short with respect to the time it takes for the satellites to move through the covariance volume, the collision path may be assumed a straight line. Since satellite position is the quantity of interest in that case, the covariance volume for estimating the location of an object is the  $3 \times 3$  position submatrix of the full covariance. These concepts are described in ANSI/AIAA S-131-2010.

When the duration of the encounter is comparable to or greater than the distance satellites move in a unit time, the collision path is not straight, the relative velocity cannot be assumed linear, and a more complete position and velocity submatrix is required, at least  $6 \times 6$ .

Satellite orbits and covariances are propagated or interpolated over the future interval of interest, depending on whether the orbit is state vector and covariance at the initiation time or whether the orbit data are ephemerides and covariances already determined at time increments over the interval of interest. The probability of collision is determined at each time increment.

The complex mathematical process of determining whether the covariance volumes of two objects touch or intersect and the methods for determining the volume of the intersection are described in normative and informative documents. The process reduces to combining the covariance volumes of both objects in the direction of the relative velocity between the objects and determining the volume contained within a cylinder whose cross section is the combined areas of both objects. Figure 2 depicts the geometry of the problem.

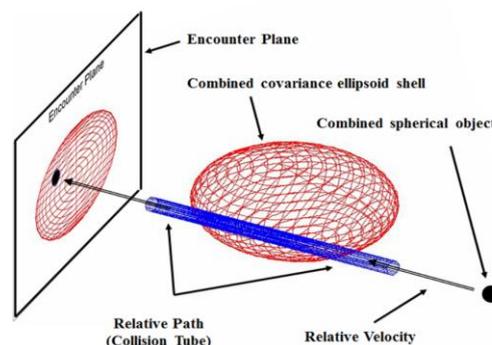


Figure 2 — The collision estimation problem

The process depicted is valid when the rate at which the encounter occurs is small compared to the relative velocity. The collision tube can be assumed linear. When the encounter occurs over a long time compared to that in which the object would move a distance comparable to the longest dimension of the covariance volume, the collision tube cannot be assumed to be straight. Bending must be accommodated consistent with the change in relative orbit curvature of one of the objects relative to the other over the encounter interval. This is the case for conjunctions among geostationary objects and conjunctions in other orbital regimes having slow closing velocity with respect to orbital velocity.

The covariance ellipsoid can be reduced to a sphere by normalizing its dimensions by the variance in each orthogonal axis. This is called Mahalanobis space. Since all cross sections are affine, scaled transformations of a circle, the problem is reduced to determining an area in a two-dimensional space. Informative references describe the formalism.

In the two-dimensional reduction, the collision probability is

$$P = \frac{1}{2 \cdot \pi \cdot \sigma_x \cdot \sigma_y} \int_{-OBJ}^{OBJ} \int_{-\sqrt{OBJ^2-x^2}}^{\sqrt{OBJ^2-x^2}} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x-xm}{\sigma_x} \right)^2 + \left( \frac{y-ym}{\sigma_y} \right)^2 \right] \right\} dy dx \quad (1)$$

where

- $OBJ$  is the combined object radius;
- $x$  lies along the minor axis;
- $y$  lies along the major axis;
- $xm$  and  $ym$  are the respective components of the projected miss distance;
- $\sigma_x$  and  $\sigma_y$  are the corresponding standard deviations.

There are several numerical techniques for determining the volume whose value is the collision probability. The mathematical statement is well documented in communication and signal detection theory. The most widely used numerical approximations to this integral are due to Foster, Chan, Patera, and Alfano. These have all been evaluated over wide ranges of governing parameters (miss distance, variances, object sizes, covariance aspect ratios).

## 6.4 Maximum probability

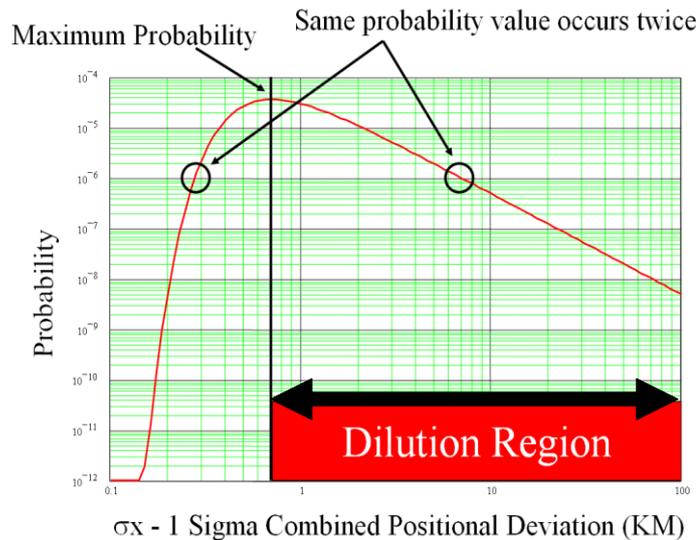
A significant amount of information is required in order to estimate the probability that two satellites might collide. This includes the external architecture of the satellite, its attitude, and specific characteristics of both the osculating orbit and the uncertainty in that orbit. Much of this is not available realistically, and it might be infeasible to seek it in a reasonable amount of time. There are two approaches to mitigate this uncertainty while still developing meaningful and trustworthy measures of risk. The first is maximum probability.

Trustworthy and realistic covariances are the essence of probability estimates. There are many reasons for covariances not being trustworthy or realistic. For example, the observations from which orbits are determined might be correlated as a result of tracking procedures. Much of the orbit uncertainty will be suppressed artificially. Process models may be deficient or the essential matches among observation frequency, mathematical sampling, physical approximations, and numerical procedures may be faulty.

It is well known that the joint probability that two objects occupy the same location in phase space has a maximum as a function of covariance dimensions. Physically, if the two orbits have been estimated precisely, it is extremely unlikely that the satellites would collide for separations greater than the sum

of both cross-section dimensions. Conversely, if the orbits are not very precise, the objects could be anywhere within large volumes, and the probability that they were in the same place is small.

Figure 3 demonstrates maximum probability in a representative situation. There is a unique value of combined covariance for which the probability is a maximum and a corresponding unique mean separation between the satellites. Note that the actual probability decreases dramatically on either side of the maximum. Therefore, the maximum probability is always very conservative. In the dilution region, probabilities decrease because we are very uncertain as opposed to the small probabilities before the maximum, which occur because we are certain where the satellites could be.



**Figure 3 — Maximum probability and dilution**

## 6.5 Bounding volume based on probability

An alternative to mitigating lack of information is the exhaustive and methodical development of a straightforward bounding volume that encompasses as much of the high-probability collision events as is reasonable. CNES excels at this approach. This technique must be applied to every satellite of interest and is most practical when an operator is responsible for only a few satellites. However, once an interested and responsible operator has determined the appropriate bounding volume for his satellites, that volume could be shared and employed whenever other observers and providers consider that satellite.

Figure 4 demonstrates the bounding volume determined for the Jules Verne automated transfer vehicle (ATV) based on extensive synthesis of collision circumstances. Table 1 demonstrates that a large, conservative bounding volume has both a high rate of detection for high-probability collisions and a correspondingly high rate of false alarms. Conversely, a smaller volume might have a low probability of detection but also a low probability of false alarms. Generally, operators are well advised to be conservative rather than risk missing potentially catastrophic events.

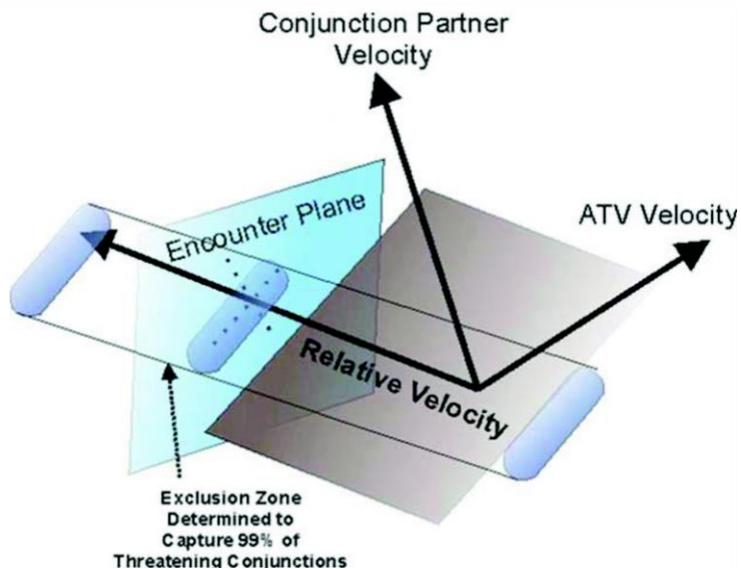


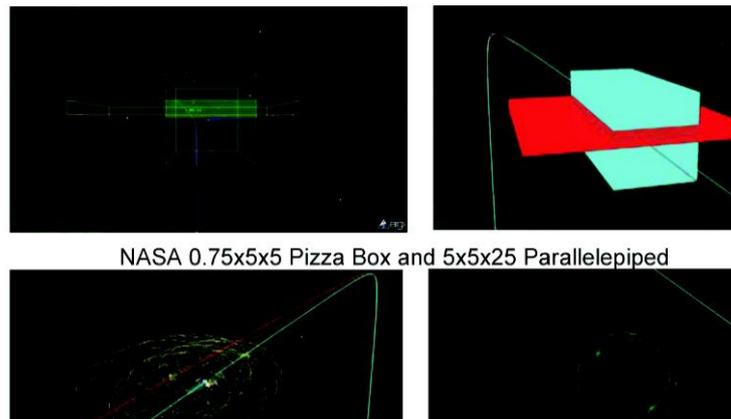
Figure 4 — Automated transfer vehicle exclusion zone

Table 1 — Probabilities of detection and probabilities of false alarm for different bounding volumes

USAF catalog number	11332		26847		26063	
	Probability of detection	Alerts per year	Probability of detection	Alerts per year	Probability of detection	Alerts per year
3 km sphere	0,44	0,2	0,24	0,3	0,08	0,7
10 km sphere	0,86	5,5	0,63	3,7	0,23	4,9
10 × 25 × 10 km box	0,92	3,6	0,78	6,7	0,28	10,1
NASA "pizza box" 0,75 × 25 × 25 km box	0,98	0,4	0,93	0,4	0,33	1,4
NSA "hockey puck" ± 5 × 30 km	0,99	3,6	0,94	5	0,37	7,5
ATV-CC ± 30 × 5 km area	1	3,6	0,99	5	0,39	7,5
USSTRATCOM ± 19 × 40 × 40 km box	1	7,6	0,97	9,8	0,42	11,1

### 6.6 Comparison of techniques

Each assessment and collision probability technique will lead to a different outcome. Figure 5 illustrates the possibilities for a real conjunction between AMC-11 and XM-3, 29 Jan 2011, 10:35 UTC.



**Figure 5 — Comparison of different screening and assessment techniques**

Figure 5 demonstrates that each screening and analysis technique will perceive events differently. The chart includes the so-called NASA Pizza Box ( $0,75 \times 5 \times 5$  km parallelepiped), a  $5 \times 5 \times 25$  km parallelepiped, covariance ellipsoids and a 3 km diameter sphere. The bounding value is centred on one of the satellites. Some perceive the close approach of one satellite to other as a threat, some do not.

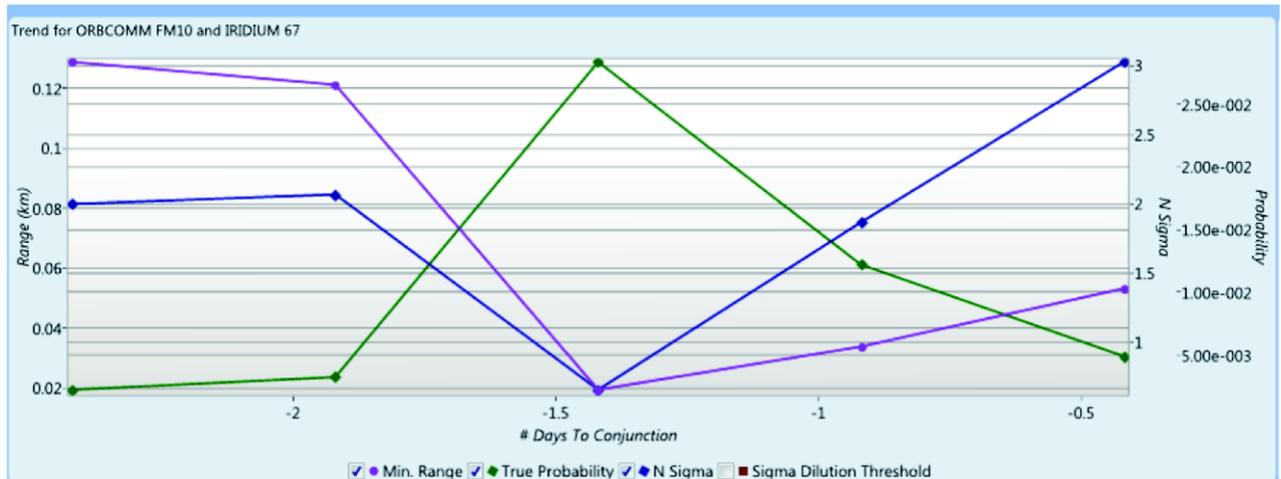
The differences in screening and assessment approaches make it necessary that those who receive warnings also be informed of the screening and assessment techniques that led to the warning.

## 7 Probability of survival

The goal of the analysis to avoid collisions is that the satellite of interest survives the estimation time interval. The highest probability collision or the one with the minimum separation distance over the time interval generally are not the only conjunctions. Operators wish their satellites not to experience any collisions, and there is a probability that each conjunction might lead to a collision. As orbit estimates evolve with new observations, close approach geometry and epoch will change. The closer the estimated epoch is to the estimated time of closest approach, the more accurate the estimate. Close approaches, even those with notable probability of collision, estimated to occur weeks from the estimated epoch hence almost never materialize.

### 7.1 Trending

Trending is following the progress of close approach between two satellites over the time interval of interest. Figure 6 is an example of the evolution of such a conjunction. The trend that a close approach distance exhibits over the estimation interval indicates decreasing separation; hence, reason for concern. Probability of collision may increase or decrease. Increasing probability of collision and decreasing separation are a cause for concern and preventive action. It is very important to understand that a single discriminant is seldom sufficient for a confident assessment.



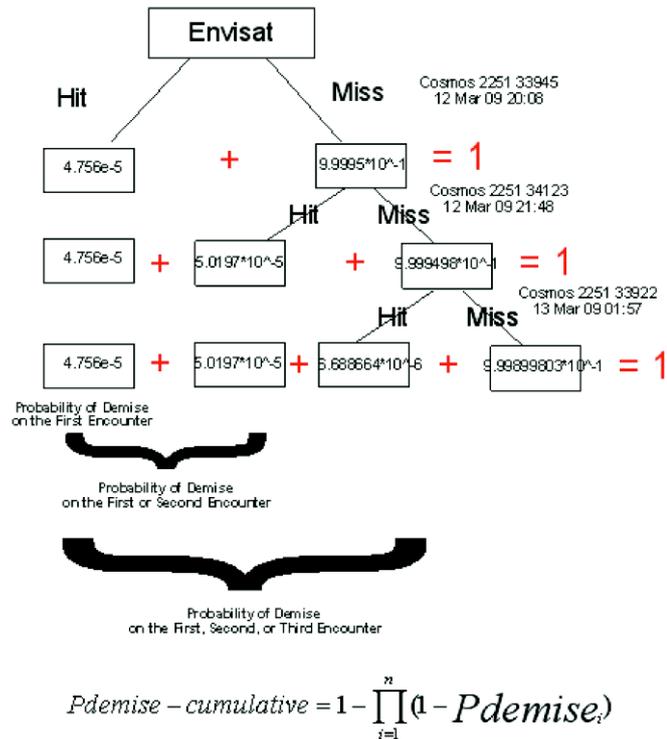
**Figure 6 — Trend of close approach between two satellites**

In addition to the short-term trending of conjunction miss distance associated with a single conjunction event, satellite operators can also minimize collision risk via monitoring and long-term trending of multiple close approach events for all pairings of their operational satellites with each other and with the rest of the orbital population. This is especially effective in the GEO regime or in constellations having common altitude ranges, where recurring close approaches can signal a long-term collision threat.

Conjunction assessment and collision avoidance require continuous vigilance for near-term events that might require unanticipated maneuvers and long-term monitoring for numerous close approaches that can be mitigated by collaborative stationkeeping among those who occupy the same assigned longitudinal slot.

## 7.2 Cumulative probability

The principle of cumulative probability accrues the probability that a single satellite will survive the analysis time period subject to all close approaches that it might experience in that interval. Each close approach taken in the order that they occur has a probability that a collision will occur and its complement, the probability that there will be no collision. If the satellite survives the first encounter, there are corresponding probabilities of demise or survival for the next encounter, and so on. Figure 7 demonstrates this chain for a real satellite in the past.



**Figure 7 — Cumulative probability hierarchy**

The sum of possibilities after each successive encounter must be unity, since the satellite will have survived or not. The process at each stage reveals the probability that the satellite would have survived one, two, or more of a sequence of encounters. These could be successive encounters with the same object over time.

It is possible that the cumulative probability of demise over several successive encounters might exceed the threshold of concern even though none of the individual encounters might have individual probability of collision above threshold.

The current threat is not the only threat, and a threat far in the future is not as credible as a threat near at hand.

### 7.3 Bayesian assessment

Bayesian assessment exploits the fundamental principles of conditional probability and multi-discriminant signal detection. Bayesian concepts systematically assess the probability that a given outcome is associated with a set of observables. The observables are called discriminants. The discriminants may be physical observables such as minimum close approach separation between two satellites, the largest probability of collision over the analysis period, or the greatest uncertainty in each satellite orbit. There may also be subjective discriminants such as whether the satellite is maneuverable or indications of the consequence of the collision, such as the amount of energy stored within the satellite. Some discriminants are explicitly quantitative. Others may be quantified subjectively. One example is associating a weight with the fact that the satellite often has close approaches that confidently have not led to collisions. The relationships among outcomes and discriminants may be analytical or implicit based on well-founded empirical beliefs. There is a significant body of research and literature. One disadvantage of beliefs is that, although the statistical formalism can confirm the connections, the physical details of the connections are not exposed. Therefore, such techniques might be very good indicators of the risk of a conjunction being significant, but they do not necessarily reveal why or provide guidance for mitigation.

## **8 Additional information for judging courses of action**

Courses of action that are available depend on more information than just close approach distance. Sometimes the only course of action or even the best is just to wait and try to mitigate consequences if the collision itself is unavoidable.

### **8.1 Maneuver capability**

Whether one or both conjunction partners can maneuver is very important. However, this itself may not be a deciding factor. Maneuvers consume propulsive energy that is intended for orbit or attitude adjustment or for safe disposal at mission end. Adding additional propellant diminishes useful payload mass. Unanticipated maneuvers can diminish mission capability and duration. Near mission end, there might not be sufficient stored energy to maneuver, but the consequences of a collision might be confidently minor. Operators must consider many factors beyond just maneuver capability in determining a course of action.

### **8.2 Spacecraft characteristics**

Spacecraft size, geometry, and the ability to adjust attitude with minimal energy expenditure must be considered. Large spacecrafts likely have large solar panels. Most of the cross section might have low areal density, which is less likely to fragment but more likely to remain in orbit. Spacecrafts such as the ISS have large overall dimensions but many voids, although it is risky to hope that another spacecraft would fly through a void, missing the satellite. Nonetheless, the overall probability of collision might account for voids.

### **8.3 Quality of underlying orbit data**

Not all orbit data are equally useful or trustworthy. The quality and credibility of orbit information even from the same provider can vary depending on the sensors that provide observations, the frequency and density of those observations, the correlations among observations as a result of data processing at the source, and even the volume of diverse observations of different satellites, burdening observational resources. The provenance of the data are embodied in the metadata that must accompany the quantitative information. This is a mandatory element of standard orbit data messages, as in ISO 26900.

## **9 Consequence assessment**

All collisions must be avoided if possible. There are so many qualifying conjunctions that all cannot be acted upon simultaneously or that actions cannot be accomplished as rapidly as possible. Even if response can be expeditious, maneuvers to avoid collisions change the orbital landscape, possibly jeopardizing satellites that were not initially involved. Restoring the original orbit will also consume energy and change the on-orbit traffic patterns. Therefore, there should be a mechanism for prioritizing responses.

### **9.1 Guidance for population risk**

The short-term (hours to weeks) evolution of debris is the greatest operational risk.

There are several models of the long-term evolution of the debris environment. The principal schemes such as Master and Evolve have been compared as noted in informative references. Others, such as PODEM, recognize nonlinearities that lead to a more threatening population but no exponential catastrophe. These models are excellent guidance for the initial stages of a mission, but they do not address the near-term threats.

There are simulations of debris production and near-term evolution of the fragments into the resident space catalogue. These are immature but they provide broad guidance for the consequences of fragmentation over periods of hours to weeks. The outcomes depend upon assumptions of the degree to which the mass of each collider is intimately involved in the collision. Without knowledge of the satellite architectures and the orientations at the instant of collision, reasonable assumptions of degree of involvement are based on the size of each and general understanding of the existence of appendages. Table 2 is an example of the near-term risk to other satellites as a result of the Cosmos 2251-Iridium 33 debris early in the aftermath.

**Table 2 — Subsequent risk associated with debris from the Cosmos 2251-Iridium 33 collision**

Satellite	Fraction of mass involved (%)	Collision partner	Conjunction epoch	Fragments created
Iridium 0610	10	Cosmos Debris (Catalog 34015)	11 Mar 09 00:24 UTC	198
Cosmos 1867	5	Cosmos Debris (Catalog 34054)	11 Mar 09 10:24 UTC	278
Fedsat	50	Iridium Debris (Catalog 34105)	13 Mar 09 03:18 UTC	68
Cosmos	5	Iridium 33 Debris (Catalog 33950)	13 Mar 09 13:20 UTC	278
Envisat	2	Cosmos Debris (Catalog 3370)	14 Mar 09 08:01 UTC	626

The table delineates each of several probable collisions, indicating the satellites and debris involved and the degree of contact between colliders at the instant of collision. The estimated number of fragments from these encounters is listed. In some cases, there were probable tertiary collisions.

These estimates and warnings of potential secondary or tertiary events could be included in information exchanges.

## 9.2 Traffic impacts

Maneuvers necessary to avoid or mitigate collisions cannot be executed spontaneously or capriciously. Considerations include energy required to evade and return to mission orbit, satellites that might be encountered during the maneuver and thereafter, and consequences of conjunctions that might be suffered as a result of maneuver. Maneuver timing is critical. Maneuvering as early as possible should be most energy efficient and safe. Discrepancies in executing the maneuver can be corrected in due course. However, orbit phasing with ground station contact and other practical matters might delay executing maneuvers. Evasive maneuvers might be combined with or influence regular stationkeeping maneuvers. Maneuvers for any reason should be screened against the resident environment to ensure that collision risks are accommodated both while executing maneuvers and thereafter.

## 10 Requirements for warning and information for avoidance

The previous discussion leads to documentary and operational requirements for warning and providing information for avoiding collisions.

### 10.1 Orbit data

It is obvious that complete orbit data are required for each satellite that is involved in the estimated conjunction. This is essential to plan mitigations and accommodate consequences. The form and format for exchanging orbit data are in ISO 26900. The decision is whether this should be included in the

conjunction warning or whether it is easily accessible otherwise, either stored and maintained current, or transmitted under separate cover. Which is best is an operator and provider collaborative decision. Any orbit data and metadata must be in standard ISO/CCSDS orbit data message configuration (see ISO 26900).

## 10.2 Minimum data required for warning of and avoiding collisions

The irreducible minimum content is as follows. Each data element is justified in terms of what is needed for.

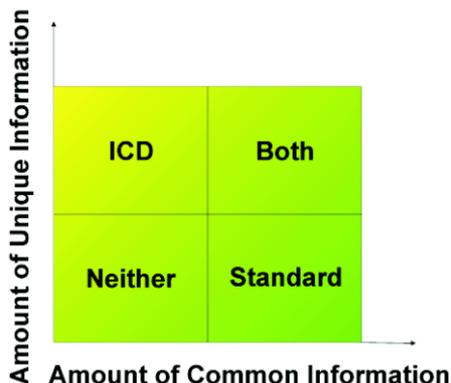
- Time of closest approach in a standard time scale. Required to determine remaining reaction time.
- Identities of the satellites involved and their operational status if known. Required for assessing consequences and mitigation opportunities.
- Closest approach distance between the two affected satellites in a standard reference frame and coordinate system. Required for assessment if orbit data are not available for each object. Otherwise can be computed knowing the estimate time of closest approach.
- A  $(6 \times 6)$  covariance matrix of three-dimensional position and velocity for both objects in well-defined reference frame at the time of closest approach if available. Required to determine collision probability.
- State of each satellite at the time of closest approach expressed either as a state vector of a single ephemeris in a standard or well-defined orbit determination and propagation scheme. Required to assess consequences and develop maneuvers that can be developed by propagating each satellite to closest approach.
- Relative velocity at closest approach in the same reference frame and coordinate system as the closest approach distance. Required for assessing consequences and developing maneuvers, if necessary or if satellite states are not available.
- Close approach threshold, the minimum safe separation that the provider imposes expressed in the same manner as the closest approach. Required because each operator has different risk tolerance. If the reported conjunction is outside the risk threshold of a recipient, the recipient can immediately disregard it.

Object size, shape, and orientation are necessary to determine true probabilities of collision, but often these are truly unknown. This can be mitigated by using a spherical approximation whose diameter is the sum of the largest dimensions of both objects and the maximum probability of collision.

All other information required for planning reaction and assessing consequences can be derived from trustworthy orbit data.

## 10.3 Optional elements of information

Best practices are approaches that are uniformly understood and applicable. Standards codify what is common to most who contribute to the development and share a common need. Information and processes unique to a minority of users should be the subject of interface control documents between specific providers and specific recipients.

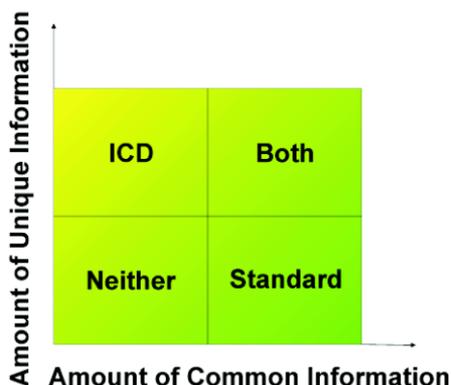


**Figure 8 — Operational execution space**

Figure 8 portrays the operational execution space. If there is considerably more information required by all who participate than there is information unique to only a few, a standard is best. If the amount of unique information far exceeds the amount of data required in common, interface control documents between each pair of participants that can provide or need the unique information are best. If little information of either type is required, no documentary or codified exchange is required. If there are large amounts of optional and mandatory content, both kinds of documents should be used. Standardized exchanges should not contain a preponderance of optional content. If most of the content is optional, that is not a standard.

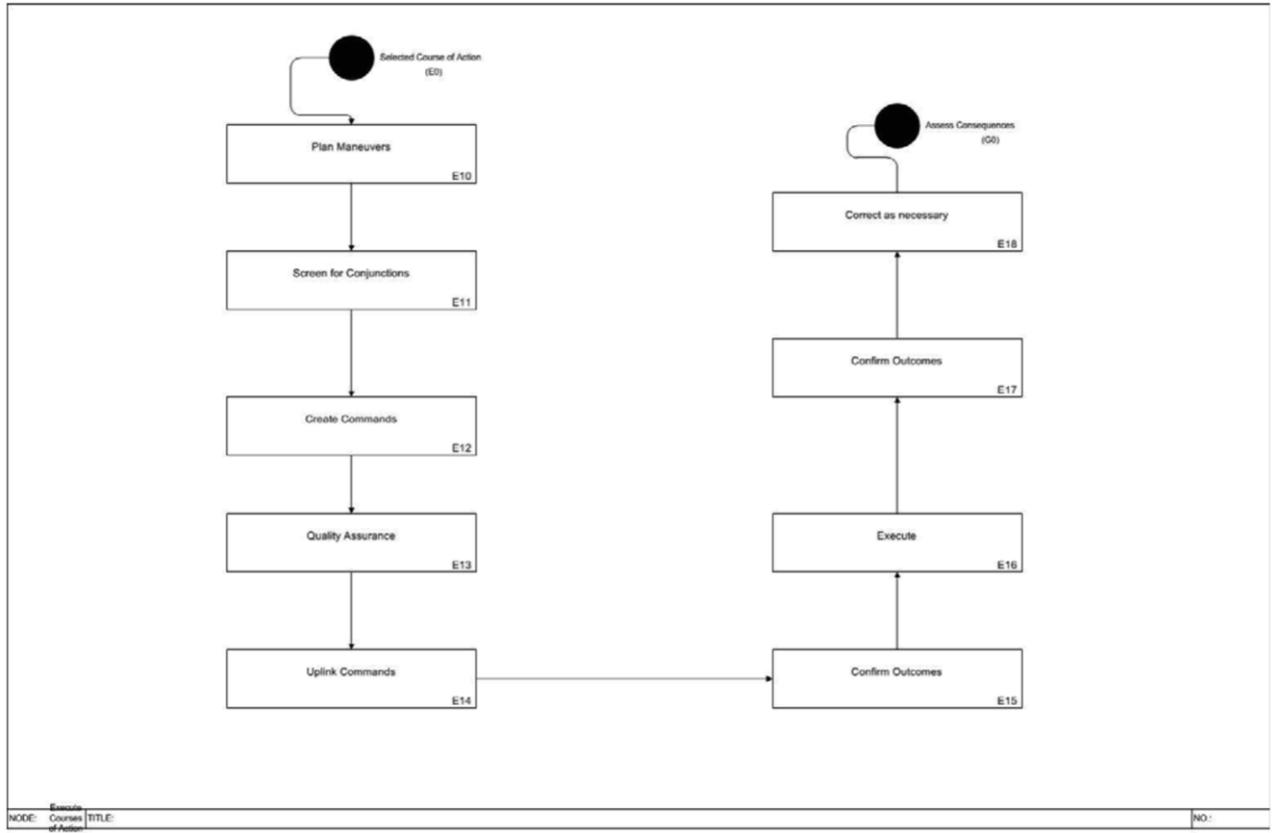
**11 Conjunction and collision assessment work flow and operational concept**

Every operation is governed by an operational concept that describes the roles, relationships, and information flows among tasks and stakeholders and the manner in which systems and processes will be used. There are several normative guides for developing and maintaining operational concepts. Since conjunction and collision assessment by definition involves multiple stakeholder, providers, and action recipients, a commonly understood, normative operational concept is essential. The following diagrams illustrate a representative operational concept.



**Figure 9 — Representative operational concept**

Figure 10 expands one of the elements of the representative operational concept.



**Figure 10 — Requirements of a function in the operational concept**

This brief exposition is to guide developing sound data requirements that enable a well-understood work flow and interactions among the potentially several organizations that must interact to mitigate the potential consequences of conjunctions and collisions.

## Bibliography

- [1] CHAN F.K., *Spacecraft Collision Probability*, The Aerospace Press, El Segundo, CA, 2007
- [2] KLINKRAD H., *Space Debris, Models and Risk Analysis*, Springer-Verlag, Berlin, 2006
- [3] SMIRNOV N.N., *Space Debris, Hazard and Mitigation*, Taylor and Francis, New York, NY, 2002
- [4] An Introduction to the Upgraded ESA Master Model, H. Sdunnus, J. Bendisch, K.D. Bunte, H. Klinkrad, P.Weger. European Conference on Spacecraft Structures, Materials, and Mechanical Testing, Dec 2000, ESASP-468, 2001, p 27
- [5] SPACE DEBRIS MODELING AT NASA. N.L. Johnson, Proceedings of the Third European Conference on Space Debris, Darmstadt, GE. *ESA SP. 2001*, **473** pp. 259–264
- [6] Comparison of Debris Environment Models: ORDEM2000, MASTER2001, and MASTER 2005, Shinya, F, Akahoshi, Y., Kitazawa, Y., and Goka, T. IHI Engineering Review, Vol 40, No 1, Feb 2007
- [7] "Low-Thrust Transfer Nomograms," Alfano, S., Paper No. AAS-18-204, AAS/AIAA Astrodynamics Specialist Conference, Snowbird, Utah, 19-23 August 2018.
- [8] "Probability of Collision: Valuation, Variability, Visualization, and Validity," Alfano, S., and Oltrogge, D., *Acta Astronautica*, Volume 148, July 2018, pp. 301-316, DOI: 10.1016/j.actaastro.2018.04.023

## Appendix-1

### Relationship between combined object size, combined positional error, and maximum probability

The individual functional relationships in the above covariance realism and hardbody radius sections indicate a joint functional association [7] between combined object size ( $r$ ), combined positional error, miss distance ( $d$ ), and maximum probability ( $P_{max}$ ) that can be expressed through the equation

$$P_{max} \cong \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{1}{1+\alpha}\right)^{\frac{1}{\alpha}} \quad (1)$$

where  $\alpha$  is

$$\alpha = \frac{r^2 AR}{d^2} \quad (AR \geq 1) \quad (6)$$

and AR is the combined covariance aspect ratio (major-to-minor axes).

We present the nomogram of Fig. 11 to illustrate these complex relationships by introducing the intermediate variable  $\beta$  such that

$$\beta = \frac{AR}{d^2} \quad (7)$$

As an example of the nomogram's use, we wish to know the maximum probability given a combined object radius of 5 m, a covariance aspect ratio of 5, and a miss distance of 5 km. Following the red arrows shown in Fig. 11, the corresponding value appears to be just under  $2.0 \times 10^{-6}$ . For comparison, the numerically-computed maximum probability is  $1.84 \times 10^{-6}$ .

With such a nomogram, it is not necessary for the user to solve algebraic equations, to look up values in data tables and possibly interpolate those values, or to use a calculator/computer to obtain results.

The user need not even have knowledge of the fundamental equation(s) or principle(s) represented. Typically, a sharp pencil and keen eye will produce results within 5% of an exact numerical solution. Nomograms not only allow fast estimation, they also can provide insight through the relationship of the various parameters. As in this case, the graphical representation allows the user to readily observe the sensitivity to changes in combined object size, covariance aspect ratio, and/or miss distance.

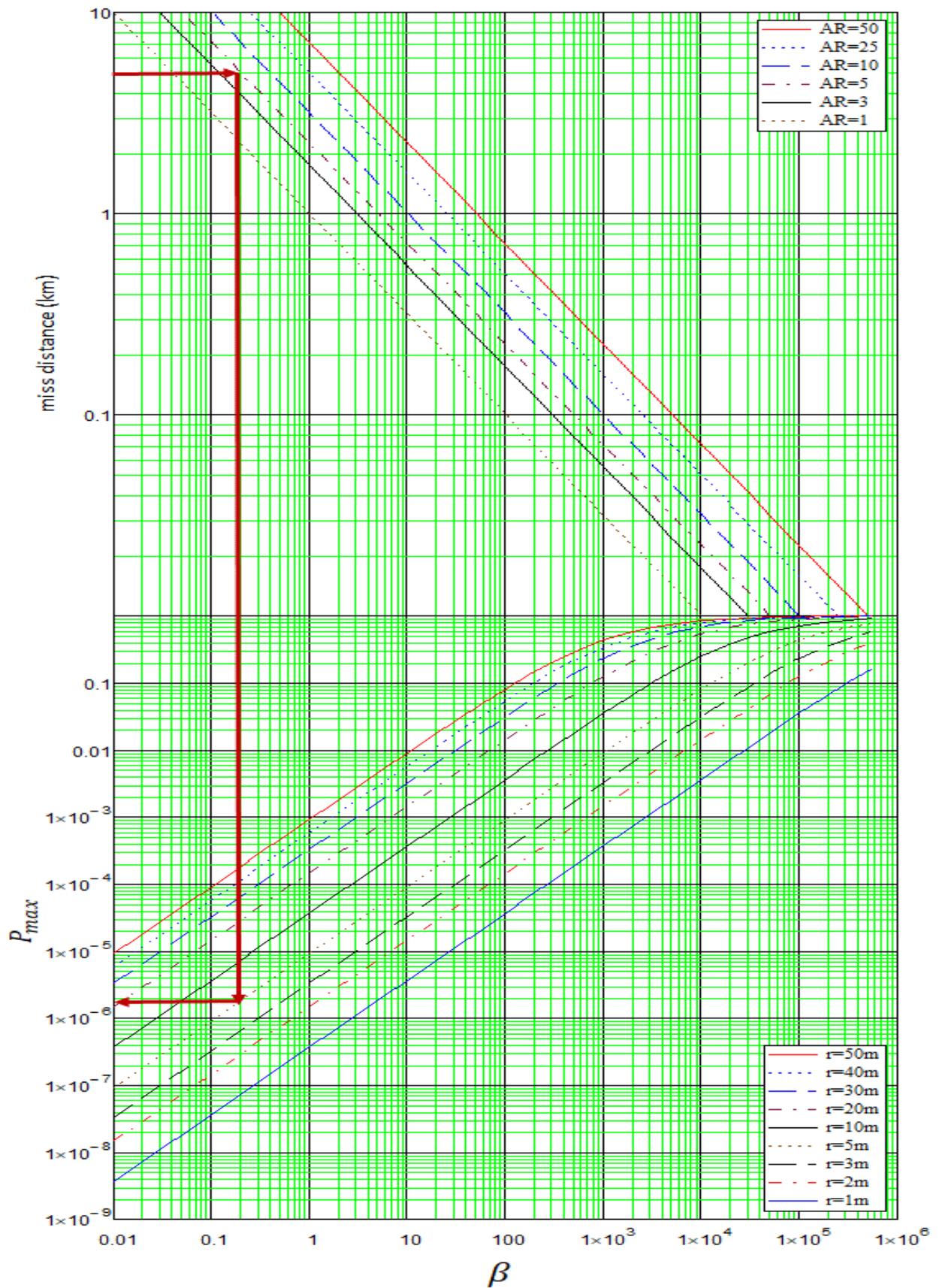


Figure 11 — Example of nomogram use to determine  $P_{max}$  from  $r$ ,  $d$ , and  $AR$

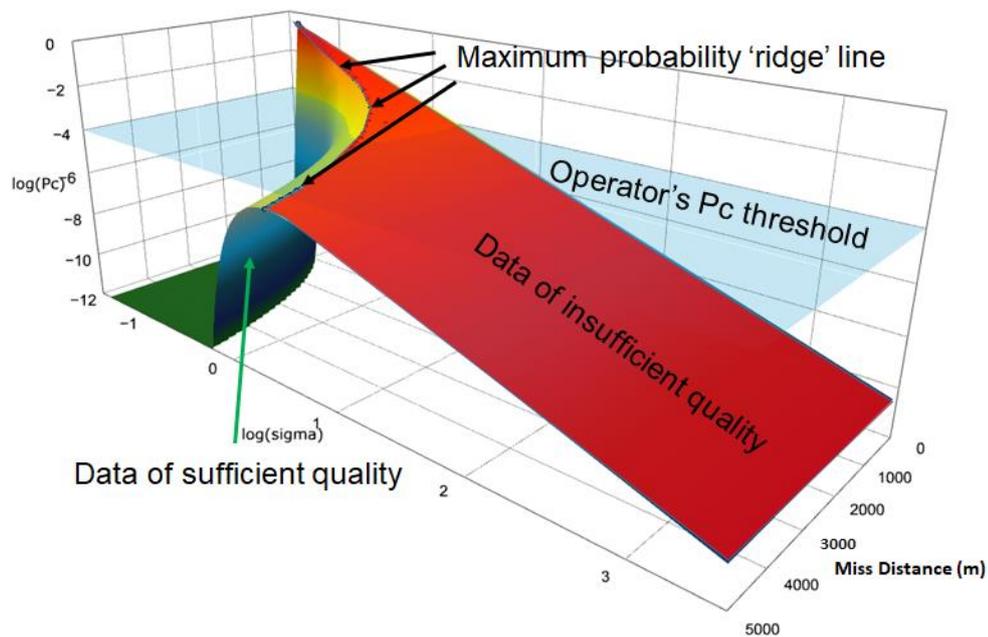
## Appendix-2

## Probability contour visualization

The approach that produced Fig. 3 can be extended to show a nominal relationship between  $P_c$  and covariance size for a range of miss distances while holding the covariance shape, orientation and combined hard body object size fixed [8]. Fig. 12 shows a topographical representation of probability contours for fixed hard body radius  $r$  in the true space. A grid is created by scaling the distance components by  $s_i$  while also scaling the standard deviations by  $C_j$ . The results are then plotted to show how probability varies with miss distances and covariance size through the following relationships:

$$P_{c_{i,j}} = P_c(r, s_i x m, s_i y m, C_j \sigma x, C_j \sigma y) \quad (8)$$

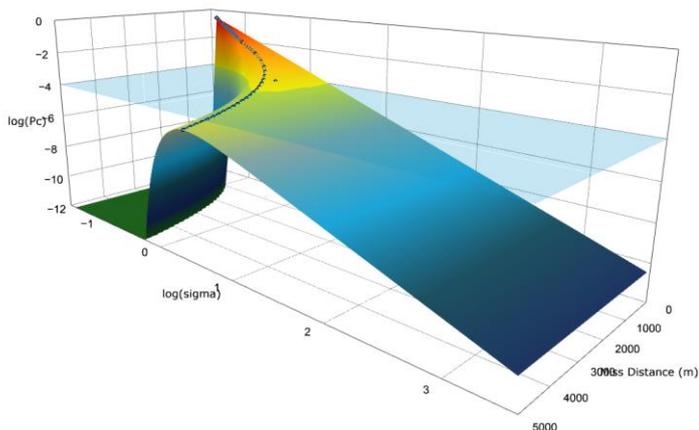
$$d_i = s_i \sqrt{xm^2 + ym^2} \quad (9)$$



**Figure 12 — Topographical representation of probability contours for fixed hard body radius**

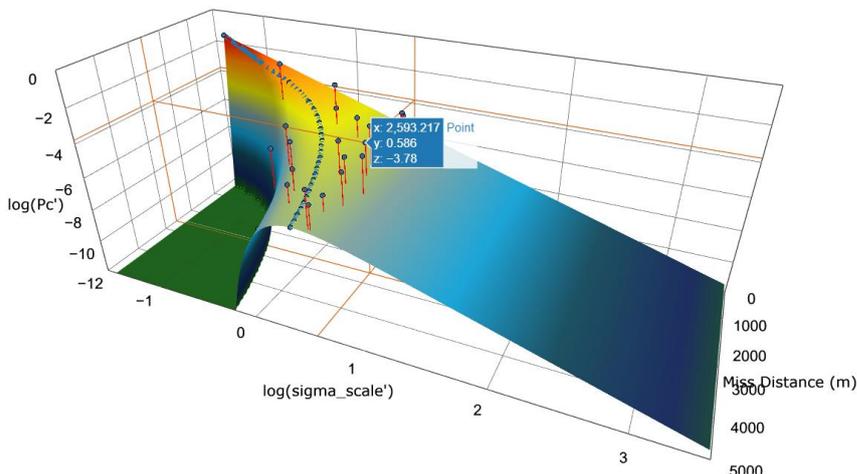
Fig. 13 was generated by an HTML script that created a three-dimensional surface plot [8]. It enables the viewer to interactively reorient the three-dimensional plot and/or zoom in/out using any browser. This HTML is created by varying  $s$  and  $C$  to create grid points through Equations (8) and (9) to generate the topography. A blue, translucent plane of constant probability is also created to define an action threshold ( $P=10^{-4}$  in this example). If below this plane and outside the dilution region (indicating that data is of sufficient quality) then no remedial action is suggested.

The topology shown in Fig. 13 is unique to all the inputs affecting the probability calculation: combined object size, miss distance components, and covariance size, shape, and orientation.



**Figure 13. Probability contours in the true space**

It is more useful to construct a common, “normalized” version of this surface projection. To accomplish this, a hybrid approach is taken to simultaneously display the estimated actual probability and its representative projection on a reference contour. In this hybrid space the estimated actual probability values (shown as dots) are accurately depicted but the contour below them is not. The surface is merely meant to show where those probabilities rest relative to the maximum probability “ridge.” The hybrid depiction is valid even with data points having different covariance aspect ratios, due to the scaling of actual probability value to the ratio of the contour’s reference maximum probability [8]. As demonstrated in the figure below, hovering the pointer over a data point reveals its information where X is the miss distance in meters, Y is log10 of covariance scale, and Z is log10 of probability. The HTML also projects that point along the orange lines so that one can see precisely where it lies on each axis. Mapping conjunctions into these spaces facilitates the examination of variations in combined object size, miss distance components, and covariance size, shape, and orientation.



**Figure 14.  $P_k$  and  $P_k'$  points displayed in the hybrid space**

Table A2-1 shows the values of some points on the hybrid space in Figure 14. This table will facilitate quick and effective conformation of collision probability relating different conditions.

Table A2-1 (draft) Values of some contour surface points of Figure 14

(1) Miss Distance: 0 to 5000 [m] z: log(Pc')

		x: Miss Distance[m]													
		0	400	800	1200	1600	2000	2400	2800	3200	3600	4000	4400	4800	5000
y: log(sigma_scale')	-1.50	-1.5589	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.45	-1.6452	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.40	-1.7342	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.35	-1.8253	-11.9756	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.30	-1.9183	-9.9809	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.25	-2.0126	-8.4170	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.20	-2.1081	-7.1953	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.15	-2.2046	-6.2455	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.10	-2.3017	-5.5115	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.05	-2.3994	-4.9491	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-1.00	-2.4976	-4.5229	-10.5986	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.95	-2.5962	-4.2049	-9.0310	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.90	-2.6951	-3.9729	-7.8064	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.85	-2.7942	-3.8092	-6.8543	-11.9294	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.80	-2.8934	-3.6997	-6.1185	-10.1498	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.75	-2.9929	-3.6333	-5.5546	-8.7568	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.70	-3.0924	-3.6011	-5.1273	-7.6709	-11.2319	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.65	-3.1920	-3.5961	-4.8084	-6.8289	-9.6575	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
-0.60	-3.2918	-3.6127	-4.5757	-6.1806	-8.4274	-11.3163	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	
-0.55	-3.3915	-3.6465	-4.4114	-5.6862	-7.4709	-9.7656	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	

Table A2-1 (draft) Values of some contour surface points of Figure 14

(1) Miss Distance: 0 to 5000 [m] (cont'd)

z: log(Pc')

		x: Miss Distance[m]													
		0	400	800	1200	1600	2000	2400	2800	3200	3600	4000	4400	4800	5000
y: log(sigma_scale')	-0.50	-3.4913	-3.6939	-4.3014	-5.3141	-6.7317	-8.5545	-10.7822	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.45	-3.5912	-3.7521	-4.2347	-5.0390	-6.1651	-7.6130	-9.3826	-11.4739	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.40	-3.6911	-3.8189	-4.2022	-4.8411	-5.7356	-6.8857	-8.2913	-9.9525	-11.8693	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.35	-3.7910	-3.8925	-4.1970	-4.7045	-5.4150	-6.3286	-7.4451	-8.7646	-10.2872	-12.0000	-12.0000	-12.0000	-12.0000	-12.0000
	-0.30	-3.8909	-3.9715	-4.2134	-4.6166	-5.1809	-5.9066	-6.7935	-7.8416	-9.0510	-10.4217	-11.9536	-12.0000	-12.0000	-12.0000
	-0.25	-3.9909	-4.0549	-4.2470	-4.5673	-5.0156	-5.5920	-6.2964	-7.1290	-8.0897	-9.1784	-10.3953	-11.7402	-12.0000	-12.0000
	-0.20	-4.0908	-4.1417	-4.2943	-4.5487	-4.9048	-5.3626	-5.9222	-6.5835	-7.3466	-8.2114	-9.1780	-10.2463	-11.4164	-12.0000
	-0.15	-4.1908	-4.2312	-4.3524	-4.5545	-4.8373	-5.2010	-5.6455	-6.1708	-6.7770	-7.4639	-8.2317	-9.0803	-10.0097	-10.5047
	-0.10	-4.2908	-4.3229	-4.4191	-4.5796	-4.8043	-5.0932	-5.4463	-5.8636	-6.3450	-6.8907	-7.5006	-8.1746	-8.9129	-9.3061
	-0.05	-4.3907	-4.4162	-4.4927	-4.6202	-4.7987	-5.0281	-5.3086	-5.6401	-6.0225	-6.4559	-6.9404	-7.4758	-8.0622	-8.3745
	0.00	-4.4907	-4.5110	-4.5717	-4.6730	-4.8148	-4.9970	-5.2198	-5.4831	-5.7869	-6.1312	-6.5160	-6.9413	-7.4071	-7.6552
	0.05	-4.5907	-4.6068	-4.6550	-4.7355	-4.8481	-4.9929	-5.1698	-5.3790	-5.6203	-5.8938	-6.1994	-6.5372	-6.9072	-7.1043
	0.10	-4.6907	-4.7035	-4.7418	-4.8057	-4.8951	-5.0101	-5.1507	-5.3168	-5.5085	-5.7257	-5.9685	-6.2369	-6.5308	-6.6873
	0.15	-4.7907	-4.8008	-4.8313	-4.8820	-4.9531	-5.0444	-5.1561	-5.2880	-5.4403	-5.6129	-5.8057	-6.0189	-6.2523	-6.3767
	0.20	-4.8907	-4.8987	-4.9229	-4.9632	-5.0197	-5.0922	-5.1809	-5.2857	-5.4067	-5.5437	-5.6969	-5.8663	-6.0517	-6.1505
	0.25	-4.9907	-4.9971	-5.0163	-5.0483	-5.0931	-5.1508	-5.2212	-5.3045	-5.4005	-5.5094	-5.6311	-5.7656	-5.9129	-5.9914
	0.30	-5.0907	-5.0957	-5.1110	-5.1364	-5.1721	-5.2178	-5.2738	-5.3399	-5.4162	-5.5027	-5.5994	-5.7062	-5.8232	-5.8855
	0.35	-5.1907	-5.1947	-5.2068	-5.2270	-5.2553	-5.2917	-5.3361	-5.3887	-5.4493	-5.5180	-5.5947	-5.6796	-5.7725	-5.8220
	0.40	-5.2907	-5.2939	-5.3035	-5.3195	-5.3420	-5.3709	-5.4062	-5.4479	-5.4961	-5.5506	-5.6116	-5.6790	-5.7529	-5.7922
	0.45	-5.3907	-5.3932	-5.4009	-5.4136	-5.4314	-5.4544	-5.4824	-5.5156	-5.5538	-5.5972	-5.6456	-5.6992	-5.7578	-5.7890
0.50	-5.4907	-5.4927	-5.4988	-5.5089	-5.5231	-5.5413	-5.5636	-5.5899	-5.6203	-5.6547	-5.6932	-5.7357	-5.7823	-5.8071	
0.55	-5.5906	-5.5923	-5.5971	-5.6051	-5.6164	-5.6309	-5.6486	-5.6695	-5.6936	-5.7210	-5.7515	-5.7853	-5.8223	-5.8420	
0.60	-5.6906	-5.6919	-5.6958	-5.7021	-5.7111	-5.7226	-5.7367	-5.7533	-5.7724	-5.7942	-5.8184	-5.8453	-5.8747	-5.8903	
0.65	-5.7906	-5.7917	-5.7947	-5.7998	-5.8069	-5.8160	-5.8272	-5.8404	-5.8556	-5.8729	-5.8922	-5.9135	-5.9368	-5.9492	

Table A2-1 (draft) Values of some contour surface points of Figure 14

(1) Miss Distance: 0 to 5000 [m] (cont'd)

z: log(Pc')

		x: Miss Distance[m]													
		0	400	800	1200	1600	2000	2400	2800	3200	3600	4000	4400	4800	5000
y: log(sigma_scale')	0.70	-5.8906	-5.8915	-5.8939	-5.8979	-5.9035	-5.9108	-5.9197	-5.9302	-5.9422	-5.9560	-5.9713	-5.9882	-6.0067	-6.0166
	0.75	-5.9906	-5.9913	-5.9932	-5.9964	-6.0009	-6.0067	-6.0137	-6.0220	-6.0316	-6.0425	-6.0547	-6.0681	-6.0829	-6.0907
	0.80	-6.0906	-6.0912	-6.0927	-6.0952	-6.0988	-6.1034	-6.1090	-6.1156	-6.1232	-6.1319	-6.1415	-6.1522	-6.1639	-6.1701
	0.85	-6.1906	-6.1910	-6.1923	-6.1943	-6.1971	-6.2007	-6.2052	-6.2104	-6.2165	-6.2234	-6.2311	-6.2395	-6.2488	-6.2538
	0.90	-6.2906	-6.2910	-6.2919	-6.2935	-6.2958	-6.2987	-6.3022	-6.3064	-6.3112	-6.3166	-6.3227	-6.3295	-6.3369	-6.3408
	0.95	-6.3906	-6.3909	-6.3917	-6.3929	-6.3947	-6.3970	-6.3998	-6.4031	-6.4070	-6.4113	-6.4161	-6.4215	-6.4274	-6.4305
	1.00	-6.4906	-6.4908	-6.4915	-6.4925	-6.4939	-6.4957	-6.4979	-6.5006	-6.5036	-6.5070	-6.5109	-6.5152	-6.5198	-6.5223
	1.05	-6.5906	-6.5908	-6.5913	-6.5921	-6.5932	-6.5947	-6.5964	-6.5985	-6.6009	-6.6037	-6.6067	-6.6101	-6.6138	-6.6158
	1.10	-6.6906	-6.6908	-6.6912	-6.6918	-6.6927	-6.6938	-6.6952	-6.6969	-6.6988	-6.7010	-6.7034	-6.7061	-6.7090	-6.7106
	1.15	-6.7906	-6.7907	-6.7911	-6.7916	-6.7923	-6.7932	-6.7943	-6.7956	-6.7971	-6.7989	-6.8008	-6.8029	-6.8053	-6.8065
	1.20	-6.8906	-6.8907	-6.8910	-6.8914	-6.8919	-6.8927	-6.8935	-6.8946	-6.8958	-6.8972	-6.8987	-6.9004	-6.9023	-6.9032
	1.25	-6.9906	-6.9907	-6.9909	-6.9912	-6.9917	-6.9922	-6.9929	-6.9938	-6.9947	-6.9958	-6.9970	-6.9984	-6.9999	-7.0007
	1.30	-7.0906	-7.0907	-7.0908	-7.0911	-7.0915	-7.0919	-7.0925	-7.0931	-7.0939	-7.0948	-7.0957	-7.0968	-7.0980	-7.0986
	1.35	-7.1906	-7.1907	-7.1908	-7.1910	-7.1913	-7.1917	-7.1921	-7.1926	-7.1932	-7.1939	-7.1947	-7.1955	-7.1965	-7.1970
	1.40	-7.2906	-7.2907	-7.2908	-7.2909	-7.2912	-7.2914	-7.2918	-7.2922	-7.2927	-7.2932	-7.2939	-7.2945	-7.2953	-7.2957
	1.45	-7.3906	-7.3907	-7.3907	-7.3909	-7.3911	-7.3913	-7.3916	-7.3919	-7.3923	-7.3927	-7.3932	-7.3937	-7.3943	-7.3946
	1.50	-7.4906	-7.4907	-7.4907	-7.4908	-7.4910	-7.4912	-7.4914	-7.4916	-7.4919	-7.4923	-7.4927	-7.4931	-7.4936	-7.4938
	1.55	-7.5906	-7.5907	-7.5907	-7.5908	-7.5909	-7.5910	-7.5912	-7.5914	-7.5917	-7.5919	-7.5923	-7.5926	-7.5930	-7.5932
1.60	-7.6906	-7.6907	-7.6907	-7.6908	-7.6908	-7.6910	-7.6911	-7.6913	-7.6915	-7.6917	-7.6919	-7.6922	-7.6925	-7.6926	
1.65	-7.7906	-7.7907	-7.7907	-7.7907	-7.7908	-7.7909	-7.7910	-7.7911	-7.7913	-7.7915	-7.7917	-7.7919	-7.7921	-7.7922	
1.70	-7.8906	-7.8907	-7.8907	-7.8907	-7.8908	-7.8908	-7.8909	-7.8910	-7.8912	-7.8913	-7.8915	-7.8916	-7.8918	-7.8919	
1.75	-7.9906	-7.9907	-7.9907	-7.9907	-7.9907	-7.9908	-7.9909	-7.9910	-7.9911	-7.9912	-7.9913	-7.9914	-7.9916	-7.9916	
1.80	-8.0906	-8.0906	-8.0907	-8.0907	-8.0907	-8.0908	-8.0908	-8.0909	-8.0910	-8.0911	-8.0912	-8.0913	-8.0914	-8.0914	
1.85	-8.1906	-8.1906	-8.1907	-8.1907	-8.1907	-8.1907	-8.1908	-8.1908	-8.1909	-8.1910	-8.1910	-8.1911	-8.1912	-8.1913	

Table A2-1 (draft) Values of some contour surface points of Figure 14

(1) Miss Distance: 0 to 5000 [m] (cont'd)

z: log(Pc')

		x: Miss Distance[m]													
		0	400	800	1200	1600	2000	2400	2800	3200	3600	4000	4400	4800	5000
y: log(sigma_scale')	1.90	-8.2906	-8.2906	-8.2907	-8.2907	-8.2907	-8.2907	-8.2908	-8.2908	-8.2908	-8.2909	-8.2910	-8.2910	-8.2911	-8.2911
	1.95	-8.3906	-8.3906	-8.3907	-8.3907	-8.3907	-8.3907	-8.3907	-8.3908	-8.3908	-8.3909	-8.3909	-8.3910	-8.3910	-8.3910
	2.00	-8.4906	-8.4906	-8.4907	-8.4907	-8.4907	-8.4907	-8.4907	-8.4907	-8.4908	-8.4908	-8.4908	-8.4909	-8.4909	-8.4910
	2.05	-8.5906	-8.5906	-8.5907	-8.5907	-8.5907	-8.5907	-8.5907	-8.5907	-8.5907	-8.5908	-8.5908	-8.5908	-8.5909	-8.5909
	2.10	-8.6906	-8.6906	-8.6906	-8.6907	-8.6907	-8.6907	-8.6907	-8.6907	-8.6907	-8.6907	-8.6908	-8.6908	-8.6908	-8.6908
	2.15	-8.7906	-8.7906	-8.7906	-8.7907	-8.7907	-8.7907	-8.7907	-8.7907	-8.7907	-8.7907	-8.7907	-8.7907	-8.7908	-8.7908
	2.20	-8.8906	-8.8906	-8.8906	-8.8907	-8.8907	-8.8907	-8.8907	-8.8907	-8.8907	-8.8907	-8.8907	-8.8907	-8.8907	-8.8908
	2.25	-8.9906	-8.9906	-8.9906	-8.9906	-8.9907	-8.9907	-8.9907	-8.9907	-8.9907	-8.9907	-8.9907	-8.9907	-8.9907	-8.9907
	2.30	-9.0906	-9.0906	-9.0906	-9.0906	-9.0907	-9.0907	-9.0907	-9.0907	-9.0907	-9.0907	-9.0907	-9.0907	-9.0907	-9.0907
	2.35	-9.1906	-9.1906	-9.1906	-9.1906	-9.1907	-9.1907	-9.1907	-9.1907	-9.1907	-9.1907	-9.1907	-9.1907	-9.1907	-9.1907
	2.40	-9.2906	-9.2906	-9.2906	-9.2906	-9.2906	-9.2907	-9.2907	-9.2907	-9.2907	-9.2907	-9.2907	-9.2907	-9.2907	-9.2907
	2.45	-9.3906	-9.3906	-9.3906	-9.3906	-9.3906	-9.3907	-9.3907	-9.3907	-9.3907	-9.3907	-9.3907	-9.3907	-9.3907	-9.3907
	2.50	-9.4906	-9.4906	-9.4906	-9.4906	-9.4906	-9.4906	-9.4907	-9.4907	-9.4907	-9.4907	-9.4907	-9.4907	-9.4907	-9.4907
	2.55	-9.5906	-9.5906	-9.5906	-9.5906	-9.5906	-9.5906	-9.5906	-9.5906	-9.5907	-9.5907	-9.5907	-9.5907	-9.5907	-9.5907
	2.60	-9.6906	-9.6906	-9.6906	-9.6906	-9.6906	-9.6906	-9.6906	-9.6906	-9.6907	-9.6907	-9.6907	-9.6907	-9.6907	-9.6907
	2.65	-9.7906	-9.7906	-9.7906	-9.7906	-9.7906	-9.7906	-9.7906	-9.7906	-9.7906	-9.7907	-9.7907	-9.7907	-9.7907	-9.7907
	2.70	-9.8906	-9.8906	-9.8906	-9.8906	-9.8906	-9.8906	-9.8906	-9.8906	-9.8906	-9.8906	-9.8907	-9.8907	-9.8907	-9.8907
	2.75	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9906	-9.9907	-9.9907	-9.9907
	2.80	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0906	-10.0907	-10.0907
	2.85	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906	-10.1906
2.90	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	-10.2906	
2.95	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	-10.3906	
3.00	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	-10.4906	
3.05	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	-10.5906	

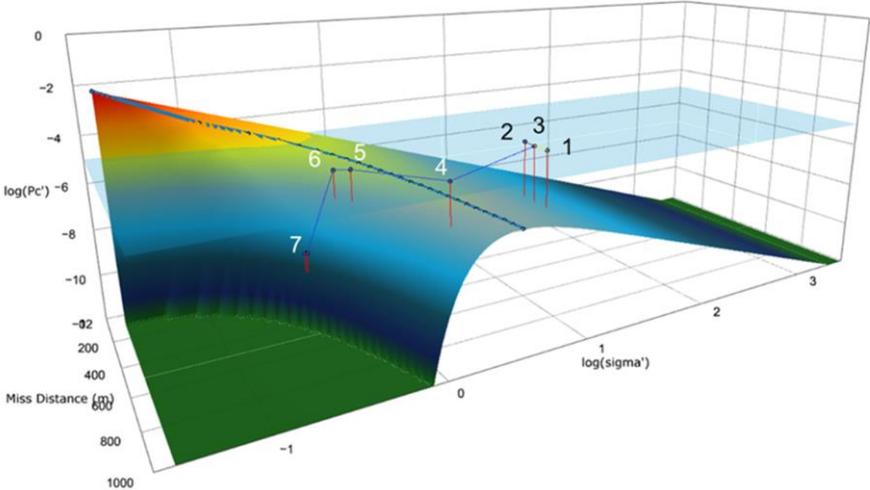
Table A2-1 (draft) Values of some contour surface points of Figure 14

(1) Miss Distance: 0 to 5000 [m] (cont'd)

z: log(Pc')

		x: Miss Distance[m]													
		0	400	800	1200	1600	2000	2400	2800	3200	3600	4000	4400	4800	5000
y: log( $\sigma_{scale}$ )	3.10	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906	-10.6906
	3.15	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906	-10.7906
	3.20	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906	-10.8906
	3.25	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906	-10.9906
	3.30	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906	-11.0906
	3.35	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906	-11.1906
	3.40	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906	-11.2906
	3.45	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906	-11.3906
	3.50	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906	-11.4906

Such a visualization tool also allows an analyst to project and examine either a time sequence, or a filtered set of samples (for example, all LEO conjunctions over the past year) of conjunction probabilities on to a common surface. These depictions indicate the usability (soundness) of data feeding a conjunction screening process. Comparing the probability predictions from different sources and epochs can be easily characterized. As shown in Figure 15, one can discover how deep into the dilution region the conjunctions are and/or examine the progression of updates relative to the maximum probability ridge line.



**Figure 15. Conjunction progression**